MK
KS
ML
AB
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WS

NAME_______________

CLASS 12MT___ or 12MTX___

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2018

YEAR 12

AP4

MATHEMATICS

Time allowed – 3 HOURS + 5 Minutes Reading Time

DIRECTIONS TO CANDIDATES:

SECTION I: Use the multiple-choice answer sheet provided.

SECTION II:

- Each question is to be commenced in a new booklet clearly marked Question 11, Question 12, etc on the top of the page.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- If you do not attempt a question, you must submit a blank booklet clearly indicating the question number, your name and class
- All questions should be placed in order - multiple-choice answer sheet followed by Question 11 to 16.
- NESA approved calculators may be used.
- The NESA Reference Sheet is provided.

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1 - 10

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1. Which of the following is a primitive of $\frac{2}{x} - sin\, x$?
   (A) $2\ \ln|x| + \cos x + C$
   (B) $2\ \ln|x| - \cos x + C$
   (C) $-\frac{2}{x^2} + cos\, x + C$
   (D) $-\frac{2}{x^2} - cos\, x + C$

2. What is the value of $\lim\limits_{x \to 4} \frac{x^2 - x - 12}{x - 4}$
   (A) 0
   (B) 4
   (C) 7
   (D) 10

3. The domain of the function $f(x) = \frac{x}{\sqrt{x+3}}$ is:
   (A) $x \neq -3$
   (B) $x > 3$
   (C) $x > -3$
   (D) $x \geq -3$

4. Which expression is a term of the geometric series $4x - 8x^2 + 16x^3 - \ldots$?
   (A) $-2048\, x^9$
   (B) $-2048\, x^{10}$
   (C) $2048\, x^9$
   (D) $2048\, x^{10}$

5. What is the equation of the tangent to the curve $y = \sin x$ at the point $(\pi, 0)$?
   (A) $y = 0$
   (B) $x - y + \pi = 0$
   (C) $x - y - \pi = 0$
   (D) $x + y - \pi = 0$

6. What is the value of k if the equation $3x^2 + kx - 4 = 0$ has -2 as one of its roots?

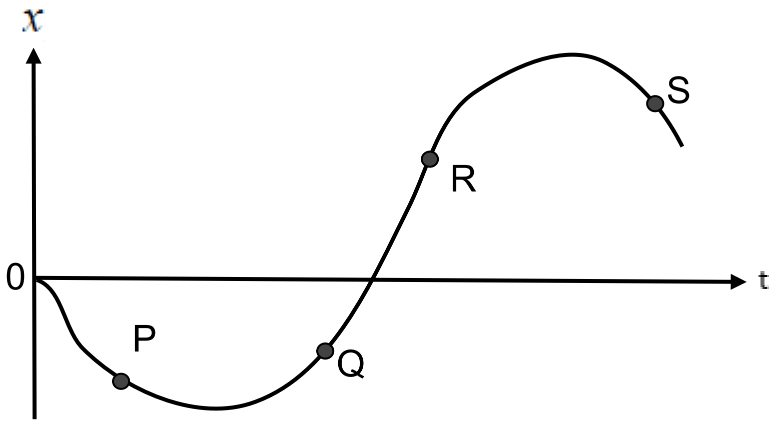
   (A) $k = 4$

   (B) $k = 8$

   (C) $k = -4$

   (D) $k = -8$

7. The displacement-time graph of a particle moving in a straight line is as shown.



At which point is the velocity negative and the particle is slowing down?

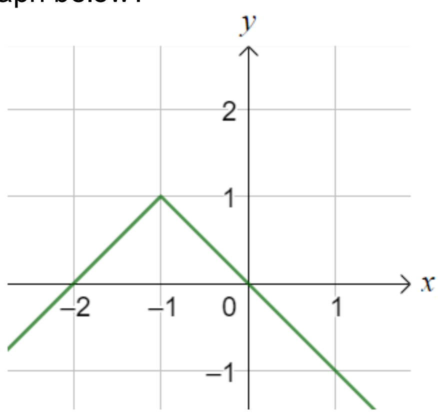
   (A) P

   (B) Q

   (C) R

   (D) S

8. What is the equation of the graph below?



   (A) $y = |x - 1| - 1$

   (B) $y = |x + 1| - 1$

   (C) $y = 1 - |x - 1|$

   (D) $y = 1 - |x + 1|$

9. Which equation represents an odd function?

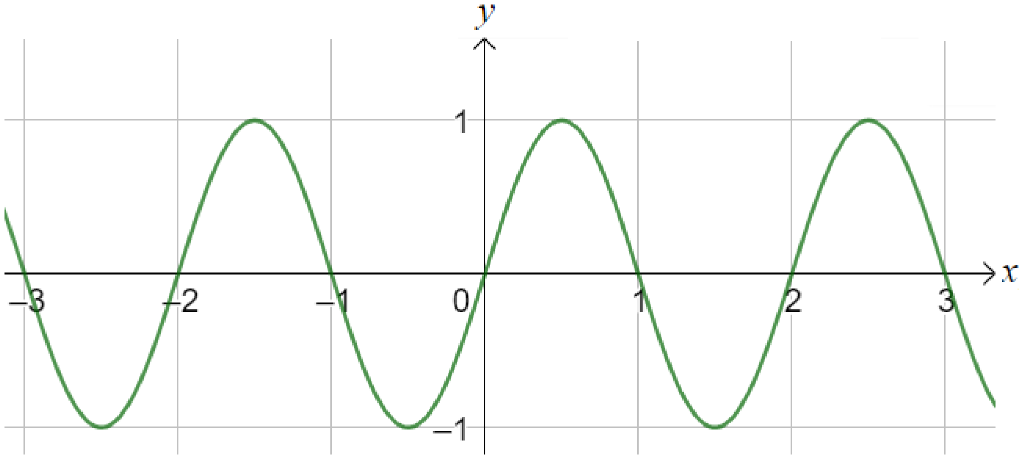
(A) $y = sin\, x + cos\, x$

(B) $y = sin\, x - cos\, x$

(C) $y = tan\, x + sec\, x$

(D) $y = tan\, x + cosec\, x$

10. The graph of the function $y = sin\,(\pi x)$ is given below.



How many solutions does $2sin\,(\pi x) + x = 0$ have?

(A) 5

(B) 3

(C) 1

(D) None of the above

End of Section I

Section II

90 marks
Attempt Questions 11 - 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet **Marks**

(a) Factorise $4x^2 + 11x - 3$ **1**

(b) If $\frac{8}{5-\sqrt{2}}$ equals $a + b\sqrt{2}$. Find the values of a and b. **2**

(c) Solve $|2x - 1| = 5$ **2**

(d) Find the exact value of $tan\ \frac{\pi}{6} + sec\ \frac{\pi}{4}$ **2**

(e) For the arithmetic sequence 31, 27, 23, 19, ….

(i) What is the 30th term? **1**

(ii) Find the sum of the first 50 terms. **1**

(f) The equation $2x^2 + 5x - 4 = 0$ has roots α and β. Find the values of

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha - 1)(\beta - 1)$ **1**

(g)

(i) Sketch the graph of $y = x^2 - 9$, showing all intercepts. **2**

(ii) Find the area between the curve $y = x^2 - 9$ and the x-axis. **1**

End of Question 11

Question 12 (15 marks) Start a new writing booklet **Marks**

(a) For the parabola $x^2 - 6x - 8y - 31 = 0$,

(i) Show that $x^2 - 6x - 8y - 31 = 0$ can be written as $(x-3)^2 = 8(y+5)$ by completing the square. **1**

(ii) Find the focal length. **1**

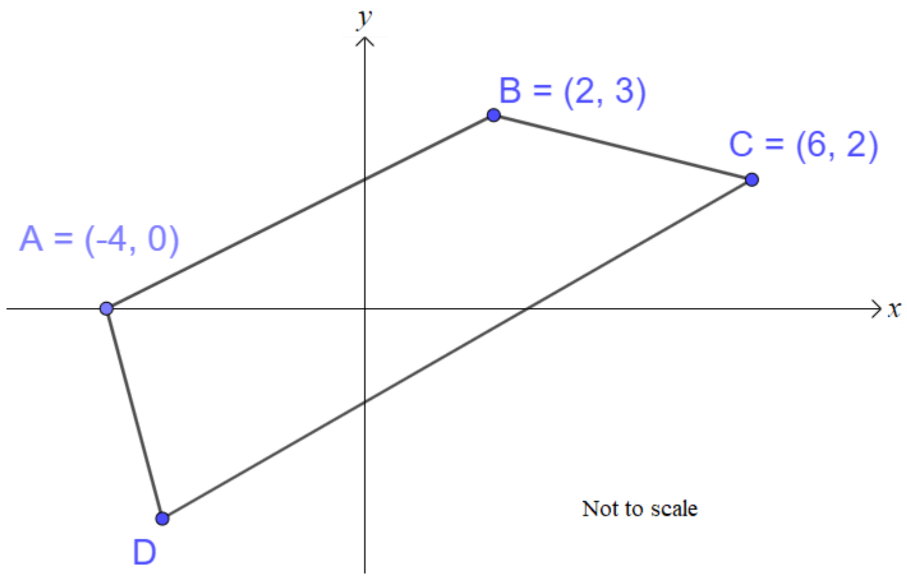
(iii) Find the coordinates of the focus. **1**

(iv) Find the equation of the directrix. **1**

(v) Find the equation of the normal to the parabola at the point $(11,3)$. **2**

(b) The graph of the function $y = f(x)$ passes through the point $(1,6)$. **2**
If $f'(x) = 4x - 5$, find $f(x)$.

(c) In the diagram, the points $A\ (-4,0), B\ (2,3), C\ (6,2)$ and D are vertices of a trapezium.



(i) Show that the equation of the line CD is $x - 2y - 2 = 0$. **2**

(ii) Given the equation of the line AD is $x + y + 4 = 0$, show that the coordinates of point D are $(-2,-2)$. **2**

(iii) Find the length AB. Give your answer in exact form . **1**

(iv) Find the perpendicular distance from point A to the line CD. **1**

(v) Find the area of the trapezium ABCD if length CD $=\sqrt{80}$. **1**

End of Question 12

Question 13 (15 marks) Start a new writing booklet **Marks**

(a) Solve $4sin^2 2x = 3$ for $-\pi \le x \le \pi$ **3**

(b) The function $y = f(x)$ is defined by $f(x) = x^3 + 6x^2 + 9x + 1$.

(i) Find any turning points and determine their nature. **3**

(ii) Find any point(s) of inflexion. **1**

(iii) Sketch the curve, showing all turning points and point(s) of inflexion. **1**

(c) The table below gives some values for $f(x) = x\, log_e\, x$

x	1	2	3	4	5
$f(x)$	0		3.33		8.05

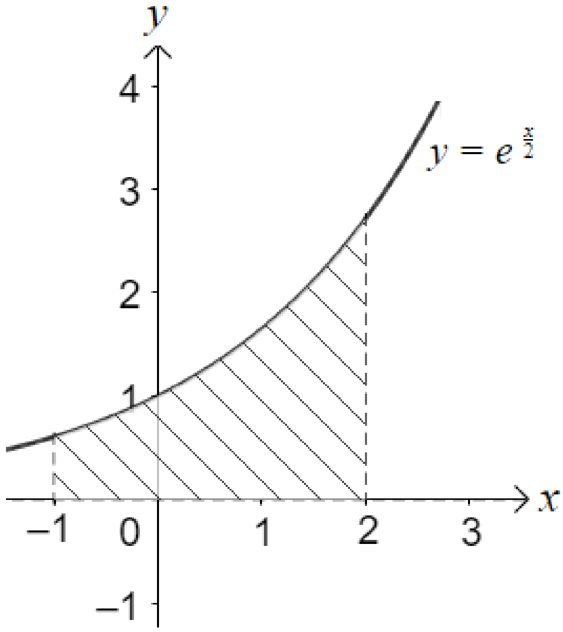
(i) Copy and complete the table of values in your writing booklet. **1**

(ii) Use Simpson's Rule with these 5 values to find an approximation of **2**

$\int_1^5 x\, log_e\, x\, dx$ correct to 2 decimal places.

(d) The region bounded by the graph of $y = x^2$, the x-axis and $x = 2$ is rotated about the x-axis. Find the volume of the solid of revolution formed. **2**

(e) The graph of $y = e^{\frac{x}{2}}$ is shown below. **2**



The shaded region is bounded by $y = e^{\frac{x}{2}}$, the x-axis, the lines $x = -1$ and $x = 2$. Find the area of the shaded region, correct to 2 decimal places.

End of Question 13

Question 14 (15 marks) Start a new writing booklet **Marks**

(a) Mary borrows \$50 000 at 6% compound interest per annum from her bank. She makes \$4000 repayments each year. The amount owed at the end of the nth year is $\$A_n$.

(i) Show that $A_2 = 50\,000(1.06)^2 - (4000(1.06) + 4000)$ **1**

(ii) Show that $A_n = 50\,000(1.06)^n - \dfrac{4000(1.06^n - 1)}{0.06}$ **2**

(iii) Find the amount of money Mary owes at the end of 12 years. Give your answer correct to the nearest dollar. **1**

(b) If $log_m a = 1.2$ and $log_m b = 0.5$, find the value of

(i) $log_m (ab)$ **1**

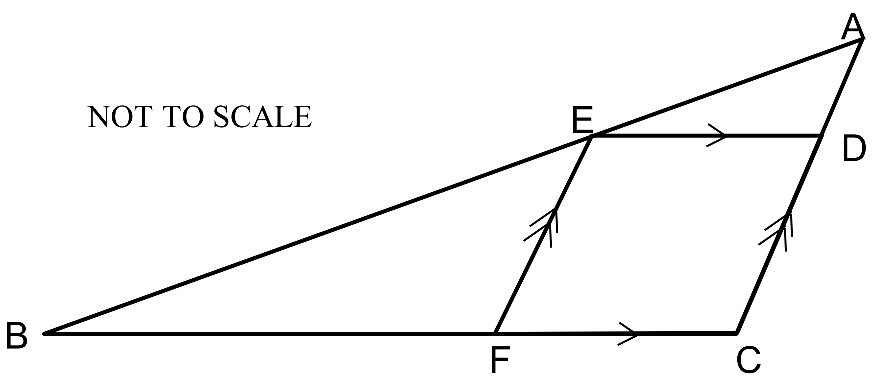
(ii) $log_m \left(\dfrac{b^3}{a}\right)$ **2**

(c)

(i) Show that $\dfrac{1}{2x-3} - \dfrac{1}{2x+3} = \dfrac{6}{4x^2-9}$ **1**

(ii) Hence find $\displaystyle\int \frac{dx}{4x^2 - 9}$ and fully simplify your answer. **3**

(d)



In the diagram above, CDEF is a rhombus inscribed inside the triangle ABC.

(i) Prove that ΔADE is similar to ΔEFB. **2**

(ii) Let the side of the rhombus be x units, length BC = 12 units and AD = 2 units. Find the value of x. **2**

End of Question 14

Question 15 (15 marks) Start a new writing booklet **Marks**

(a) A tank initially containing 500 litres of water, is leaking.

The rate at which water is leaking from the tank is given by $\frac{dV}{dt} = -\frac{1600}{(t+4)^2}$

where V is the volume of water remaining in the tank and t is the time in hours after the water started to leak.

(i) Find the volume of the water in the tank after 20 hours. **3**

(ii) How much water will be left in the tank eventually? **1**

(b) A number of frogs were introduced to an island. The population of frogs after t months is given by $P(t) = Ae^{kt}$, where A and k are constants.

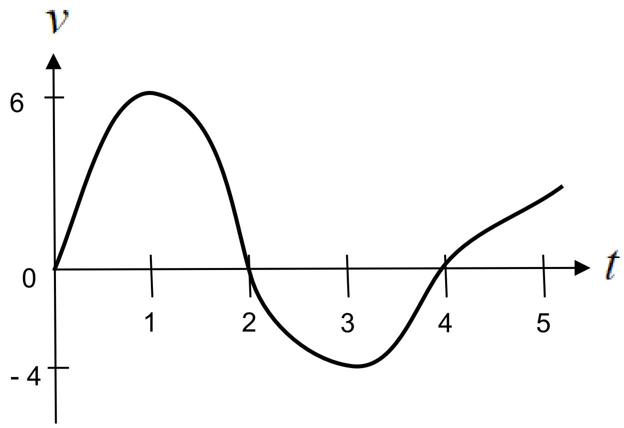
(i) Show that $P(t)$ satisfies $\frac{dP}{dt} = kP$. **1**

(ii) Five months after the frogs were introduced, their population is now 80% **more** than the initial population. **2**

Show that $k = 0.1176$ correct to 4 significant figures.

(iii) After how many months will the number of frogs first exceed one hundred times its initial population? **2**

(c)



The graph above shows the velocity v, in m/s of a particle after t seconds moving in a straight line.

(i) What is the acceleration when $t = 1$ second? **1**

(ii) When does the particle first change direction? **1**

(iii) What does the area under the curve from $t = 0$ to $t = 2$ represent? **1**

(d) A particle that is initially at rest at the origin starts moving along the x –axis.

The velocity of the particle at time t is given by $\dot{x} = 12(t - t^3)$ ms^{-1}.

(i) Once moving, when does the particle come to rest again? **1**

(ii) Find the distance travelled in the first 2 seconds. **2**

End of Question 15

Question 16 (15 marks) Start a new writing booklet **Marks**

(a) Differentiate with respect to x

(i) $tan\frac{x}{3}$ **1**

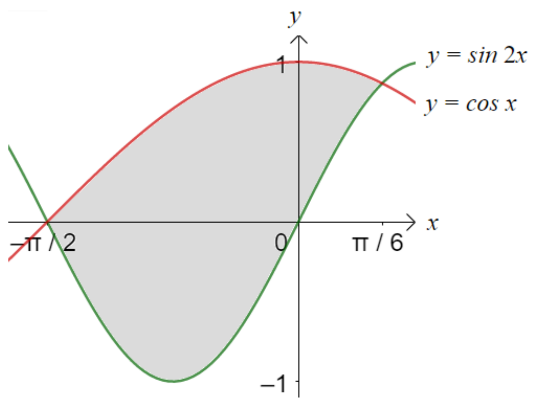
(ii) $sin^4 x$ **1**

(b) Find $\int sec^2\left(\frac{x-3}{2}\right)dx$ **1**

(c) (i) Given $y = sin\, x^2$, find y' **1**

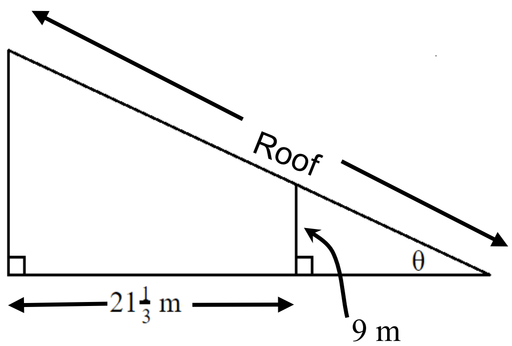
(ii) Hence, or otherwise, determine $\int x\, cos\, x^2 dx$ **2**

(d) **3**



The diagram shows the graphs of $y = cos\, x$ and $y = sin\, 2x$. The two graphs intersect at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{6}$. Calculate the area of the shaded regions.

(e)



A structure is made of two vertical walls and a roof, making an angle θ with the ground. The horizontal distance between the two walls is $21\frac{1}{3}$ metres and the height of the shorter wall is 9 metres.

(i) Show that the length of the roof is $L = \frac{9}{sin\,\theta} + \frac{64}{3cos\,\theta}$ **3**

(ii) Find the length of the shortest roof. **3**

End of Paper

HSC Mathematics

AP4 2018

NAME ____

CLASS 12 MA______________12 MTX______________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

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1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

Question	Marks
1-10	/ 10
11	/ 15
12	/ 15
13	/ 15
14	/ 15
15	/ 15
16	/ 15
Total	/ 100

CTHS MATHS 2U 2018 Trial solutions.

1) A 2) C 3) C 4) B 5) D 6) A 7) A 8) D 9) D 10) A

11) a) $4x^2+11x-3$

$=(4x-1)(x+3)$

b) $\dfrac{8(5+\sqrt{2})}{(5-\sqrt{2})(5+\sqrt{2})}$

$=\dfrac{40+8\sqrt{2}}{23}$

$=\dfrac{40}{23}+\dfrac{8}{23}\sqrt{2}$

$a=\dfrac{40}{23}$, $b=\dfrac{8}{23}$

c) $2x-1=\pm 5$

$2x=-4$ or $2x=6$

$x=-2$ $\quad$ $x=3$

d) $\dfrac{1}{\sqrt{3}}+\sqrt{2}$

$=\dfrac{\sqrt{3}}{3}+\sqrt{2}$

$=\dfrac{\sqrt{3}+3\sqrt{2}}{3}$

e) i) $a=31$, $d=27-31$

$d=-4$

$T_n=a+(n-1)d$

$T_{30}=31+(30-1)(-4)$

$=-85$

ii) $S_n=\dfrac{n}{2}[2a+(n-1)d]$

$S_{50}=\dfrac{50}{2}[2(31)+(50-1)(-4)]$

$=-3350$

$a=2$, $b=5$, $c=-4$

f) i) $\alpha+\beta=-\frac{b}{a}$

$=-\frac{5}{2}$

ii) $\alpha\beta=\frac{c}{a}$

$=\frac{-4}{2}$

$=-2$

iii) $(\alpha-1)(\beta-1)$

$=\alpha\beta-\alpha-\beta+1$

$=\alpha\beta-(\alpha+\beta)+1$

$=-2-(-\frac{5}{2})+1$

$=\frac{3}{2}$

g) i) $y=x^2-9$

y-intercept: $x=0$

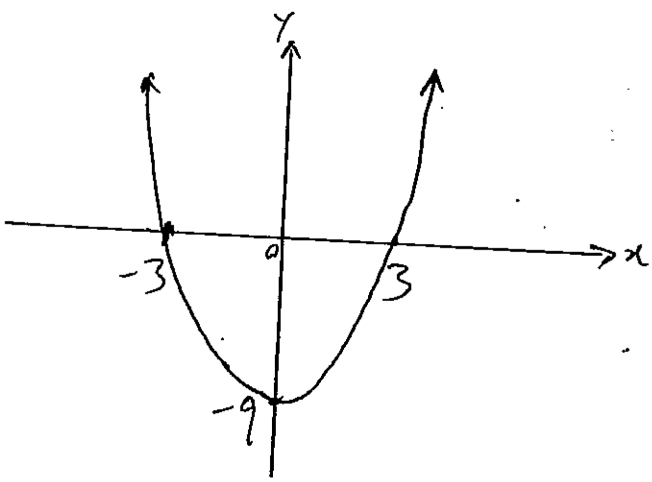
$y=-9$

x-intercepts: $y=0$

$0=x^2-9$

$x^2=9$

$x=\pm 3$



ii) $A=\left|\int_{-3}^{3}x^2-9\,dx\right|$

$=\left|\left[\frac{x^3}{3}-9x\right]_{-3}^{3}\right|$

$=36$ units^2

Alternatively,

$A=2\left|\int_{0}^{3}x^2-9\,dx\right|$

$=2\left|\left[\frac{x^3}{3}-9x\right]_{0}^{3}\right|$

$=2\left|\left[9-27\right]\right|$

$=36$ units^2

12 a) i) $x^2-6x-8y-31=0$

$x^2-6x+9-9=8y+31$

$(x-3)^2=8y+40$

$(x-3)^2=8(y+5) \rightarrow$ GIVEN

ii) $8=4a$

$a=2$

focal length $=2$

iii) Focus $=(3,-3)$

iv) Eqn of directrix: $y=-7$

$\Big\}$ cfm

v) $y=\frac{(x-3)^2}{8}-5$

$\frac{dy}{dx}=\frac{x-3}{4}$

At $x=11$,

$\frac{dy}{dx}=\frac{11-3}{4}$

$=2$

OR $y=\frac{x^2-6x+9-40}{8}$

$=\frac{x^2}{8}-\frac{6x}{8}-\frac{31}{8}$

$y'=\frac{x}{4}-\frac{3}{4}$

at $x=11$, $m_{tangent}=2$

$m_{normal}=-\frac{1}{2}$

Eqn of normal:

$y-3=-\frac{1}{2}(x-11)$

$2y-6=-x+11$

$x+2y-17=0$

b) $f'(x)=4x-5$

$f(x)=\int 4x-5$

$f(x)=2x^2-5x+c$

$6=2(1)^2-5(1)+c$

$c=9$

$\therefore f(x)=2x^2-5x+9$

c) i) $m_{CD} = m_{AB}$

$$\frac{y-2}{x-6} = \frac{3}{2-(-4)}$$

$$\frac{y-2}{x-6} = \frac{1}{2}$$

$$2y-4 = x-6$$

$$x-2y-2=0 \longrightarrow \text{GIVEN}$$

or $m_{AB} = \frac{3-0}{2--4} = \frac{3}{6} = \frac{1}{2}$

equation of CD // to AB

$$(y-2) = \frac{1}{2}(x-6)$$

$$2y-4 = x-6$$

$$0 = x-2y-2 \quad \text{GIVEN}$$

ii) Line CD: $x-2y-2=0$ —— ①

Line AD: $x+y+4=0$ —— ②

②−①: $3y+6=0$

$$y=-2$$

Sub $y=-2$ into ①,

$$x-2(-2)-2=0$$

$$x=-2$$

$$D=(-2,-2) \longrightarrow \text{GIVEN}$$

iii) $AB = \sqrt{[2-(-4)]^2+3^2}$

$= \sqrt{45}$ units

$= 3\sqrt{5}$ units

iv) $$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$= \frac{|(1)(-4)+0+(-2)|}{\sqrt{1^2+(-2)^2}}$$

$$= \frac{6}{\sqrt{5}}$$

$$= \frac{6\sqrt{5}}{5} \text{ units}$$

v) $$A = \frac{1}{\cancel{2}}(3\sqrt{5}+\sqrt{80}) \times \frac{\cancel{6}^{3}\sqrt{5}}{5}$$

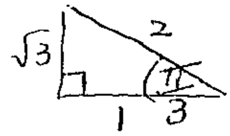
$$= (3\sqrt{5}+4\sqrt{5}) \times \frac{3\sqrt{5}}{5}$$

$$= \frac{7\sqrt{5}\times 3\sqrt{5}}{5}$$

$$= 21 \text{ units}^2$$

13 a) $4\sin^2 2x = 3 \qquad -\pi \le x \le \pi$

$\sin 2x = \pm\frac{\sqrt{3}}{2} \qquad -2\pi \le 2x \le 2\pi$



$= \frac{\pi}{3}$.

$2x = \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}, \pm\frac{5\pi}{3}$

$x = \pm\frac{\pi}{6}, \pm\frac{\pi}{3}, \pm\frac{2\pi}{3}, \pm\frac{5\pi}{6}$

$\pm 30°, \pm 60°, \pm 120°, \pm 150$

b) $f(x) = x^3 + 6x^2 + 9x + 1$

i) $f'(x) = 3x^2 + 12x + 9$

$f''(x) = 6x + 12$

Stationary points: $f'(x) = 0$

$0 = 3x^2 + 12x + 9$

$0 = x^2 + 4x + 3$

$0 = (x+1)(x+3)$

$x = -1$ or $x = -3$

$y = -3 \qquad y = 1$

$f''(-1) = 6(-1) + 12 \qquad f''(-3) = 6(-3) + 12$

$= -6 + 12 \qquad = -18 + 12$

$= 6 \qquad = -6$

$> 0 \qquad < 0$

$(-1, -3)$ is a rel minimum and $(-3, 1)$ is a rel maximum.

ii) Possible pt of inflexion: $f''(x) = 0$

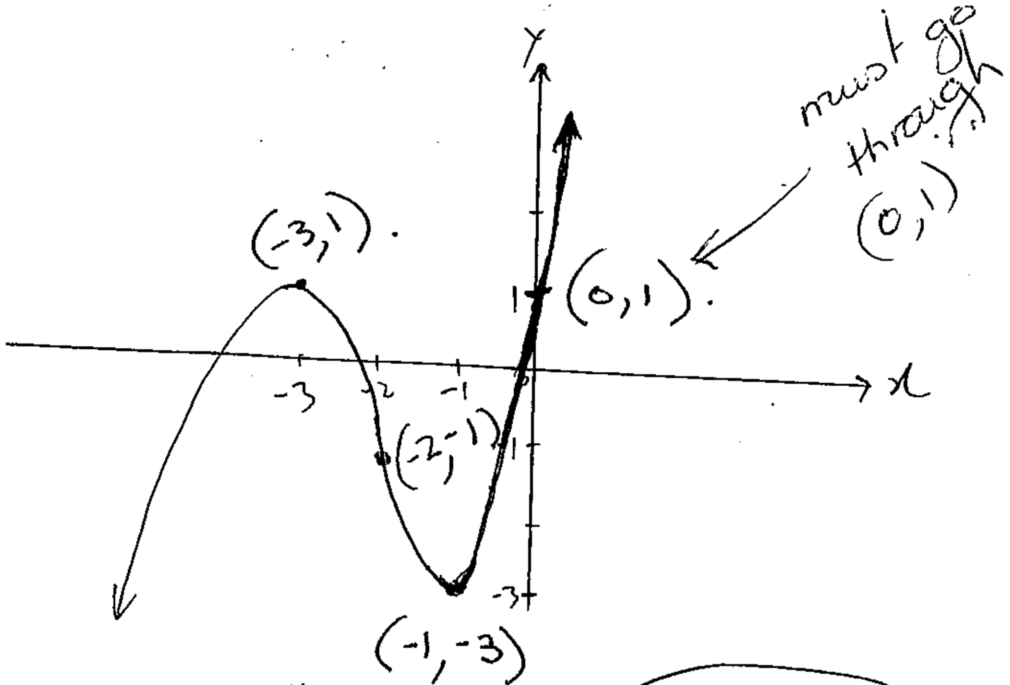
$0 = 6x + 12$

$x = -2, \ y = -1$

x	-3	-2	-1
$f''(x)$	-6	0	6

Concavity changes. $\therefore$ $(-2, -1)$ is a point of inflexion.

iii)



c) i)

	y_0	y_1	y_2	y_3	y_4
x	1	2	3	4	5
$f(x)$	0	1.39	3.33	5.55	8.05

3.3

ii) $\int_1^5 x\ln x\,dx = \int_1^3 x\ln x\,dx + \int_3^5 x\ln x\,dx$

$\approx \frac{3-1}{6}\left[0+4(1.39)+3.33\right] + \frac{5-3}{6}\left[3.33+4(5.55)+8.05\right]$

$= 14.16$

OR $\frac{1}{3} \times \frac{5-1}{4}\left[0 + 8.05 + 2(3.33) + 4(1.39 + 5.55)\right]$ 1 mark 1mk for 28.31 (cfr

d) $V = \pi\int_0^2 y^2\,dx$

$= \pi\int_0^2 x^4\,dx$

$= \pi\left[\frac{x^5}{5}\right]_0^2$

$= \frac{32\pi}{5}$ units^3 or 6.4π

$V = \pi\int_0^2 x^2\,dx$

$= \frac{8\pi}{3}$

OR 20.1

e) $A = \int_{-1}^2 e^{\frac{x}{2}}\,dx$

$= 2\left[e^{\frac{x}{2}}\right]_{-1}^2$

$= 2\left[e - e^{-\frac{1}{2}}\right] =$

~~≈ 0.79 (2d.p.)~~

$= 4.22$

Question 14

a) i) $A_1 = 50000\,(1+0.06) - 4000$

$= 50000\,(1.06) - 4000$

$A_2 = A_1 \times 1.06 - 4000$

$A_2 = [50000\,(1.06) - 4000] \times 1.06 - 4000$

$= 50000\,(1.06)^2 - 4000\,(1.06) - 4000$

$= 50000\,(1.06)^2 - (4000\,(1.06) + 4000)$ as required.

ii) $A_3 = A_2\,(1.06) - 4000$

$= [50000(1.06)^2 - (4000\,(1.06) + 4000)]\,(1.06) - 4000$

$= 50000\,(1.06)^3 - 4000\,(1.06)^2 - 4000\,(1.06) - 4000$

$= 50000\,(1.06)^3 - 4000\,(1.06^2 + 1.06 + 1)$

$A_n = 50000\,(1.06)^n - 4000\,(\underbrace{1.06^{n-1} + 1.06^{n-2} + \ldots + 1.06 + 1}_{\text{is a GP with } a=1 \quad r=1.06 \quad n \text{ terms}})$

$A_n = 50000\,(1.06)^n - 4000\left[1 \times \dfrac{1.06^n - 1}{1.06 - 1}\right]$

$= 50000\,(1.06)^n - \dfrac{4000\,(1.06^n - 1)}{0.06}$ as required.

iii) $A_{12} = 50000\,(1.06)^{12} - \dfrac{4000\,(1.06^{12} - 1)}{0.06}$

$= \$33130.06$

$= \$\,33130$ (nearest dollar)

b) i) $\log_m (ab) = \log_m a + \log_m b$

$= 1.2 + 0.5$

$= 1.7$

ii) $\log_m \frac{b^3}{a} = \log_m b^3 - \log_m a$

$= 3 \log_m b - \log_m a$

$= 3(0.5) - 1.2$

$= 0.3$

c) i) $\frac{1}{2x-3} - \frac{1}{2x+3} = \frac{1(2x+3) - 1(2x-3)}{(2x-3)(2x+3)}$

$= \frac{\cancel{2x} + 3 - \cancel{2x} + 3}{4x^2 - 9}$

$= \frac{6}{4x^2-9}$

ii) $\int \frac{dx}{4x^2-9} = \frac{1}{6} \int \frac{6}{4x^2-9} \, dx$

$= \frac{1}{6} \int \frac{1}{2x-3} - \frac{1}{2x+3} \, dx$

$= \frac{1}{6} \times \frac{1}{2} \int \frac{2}{2x-3} - \frac{2}{2x+3} \, dx$

$= \frac{1}{12} [\ln |2x-3| - \ln |2x+3| + c$

$= \frac{1}{12} \ln \left| \frac{2x-3}{2x+3} \right| + c$

In ΔADE and ΔEFB

d) i) $\angle DEA = \angle FBE$ (corresponding angles on $ED \parallel BC$ are equal)

$\angle DAE = \angle FEB$ (corresponding angles on $AC \parallel EF$ are equal)

$\therefore \Delta ADE \parallel\!\mid \Delta EFB$ (equiangular)

NOTE : Above is the most efficient solution but other solutions supported by valid reasons were accepted.

ii) $\dfrac{AD}{DE} = \dfrac{EF}{BF}$ (matching sides in $\Delta ADE \parallel\!\mid \Delta EFB$ are in proportion)

$\dfrac{2}{x} = \dfrac{x}{12-x}$

$24 - 2x = x^2$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = -6, 4$

But x is a length $\therefore x > 0$. Hence $x = 4$.

Note : Students who used the ratio

$\dfrac{2}{x} = \dfrac{2+x}{12}$

gained 1 mark if they correctly solved for x, but did not gain credit as they had not established $\Delta ADE \parallel\!\mid \Delta ACB$

15) a) i) $\frac{dV}{dt} = -\frac{1600}{(t+4)^2}$

$= -1600(t+4)^{-2}$

$V = \int -1600(t+4)^{-2}\,dt$

$= 1600(t+4)^{-1} + c$

$V = \frac{1600}{t+4} + c$

When $t=0$, $V=500$.

$500 = \frac{1600}{4} + c$

$c = 100$

$\therefore V = \frac{1600}{t+4} + 100$

When $t=20$,

$V = \frac{1600}{20+4} + 100$

$= 166\frac{2}{3}$ litres or $\frac{500}{3}$ L

ii) When $t \to \infty$,

$V \to 100$ as $\frac{1600}{t+4} \to 0$.

The volume will eventually be reduced to 100L.

b) i) $P = Ae^{kt}$

$\frac{dP}{dt} = Ake^{kt}$

$= k(Ae^{kt})$

$= kP$

ii) $P = 1.8A$ when $t=5$

$1.8\cancel{A} = \cancel{A}e^{5k}$

$\ln 1.8 = 5k$

$k = \frac{\ln 1.8}{5}$

≈ 0.1176 (4 sig. fig.)

iii) $P = Ae^{0.1176t}$

$P > 100A$

$\cancel{A}e^{0.1176t} > 100\cancel{A}$

$0.1176t > \ln 100$

$t > \frac{\ln 100}{0.1176}$

$t > 39.1596...$

$t = 40$

It will take 40 months.

39.16 or 39.2 also accepted.

c) i) $a = 0$.

ii) $t = 2$

iii) It represents the distance travelled or displacement in the first two seconds.

d) i) $\dot{x} = 0$

$0 = 12(t - t^3)$

$0 = t(1 - t^2)$

$0 = t(1+t)(1-t)$

$t = 0, -1, 1$.

The particle comes to rest again after 1 second.

ii) Distance travelled $= \left|\int_0^1 12(t - t^3)\,dt\right| + \left|\int_1^2 12(t-t^3)\,dt\right|$

$= \left|12\left[\frac{t^2}{2} - \frac{t^4}{4}\right]_0^1\right| + \left|12\left[\frac{t^2}{2} - \frac{t^4}{4}\right]_1^2\right|$

$= 12\left(\frac{1}{2} - \frac{1}{4}\right) + \left|12\left[(2-4) - \left(\frac{1}{2} - \frac{1}{4}\right)\right]\right|$

$= 3 + 27$

$= 30$ m

or Finding displacement function and constant of integration.

~~10~~/16 a) i) $\frac{d}{dx}\left(\tan\frac{x}{3}\right) = \frac{1}{3}\sec^2\left(\frac{x}{3}\right)$

ii) $\frac{d}{dx}\left(\sin^4 x\right) = 4\sin^3 x\cos x$

b) $\int \sec^2\left(\frac{x-3}{2}\right) dx = 2\tan\left(\frac{x-3}{2}\right) + c$

c) i) $y = \sin x^2$

$y' = 2x\cos x^2$

ii) $\int x\cos x^2 dx = \frac{1}{2}\int 2x\cos x^2 dx$

$= \frac{1}{2}\sin x^2 + c$

$2 \int x\cos x^2 dx$.

$\frac{1}{2}\sin x^2 + c$

d) $A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} \cos x - \sin 2x \, dx$

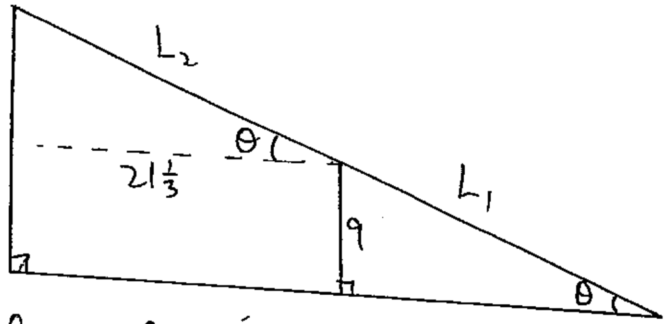
$= \left[\sin x + \frac{\cos 2x}{2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{6}}$

$= \left[\left(\sin\frac{\pi}{6} + \frac{\cos\frac{\pi}{3}}{2}\right) - \left(\sin\left(-\frac{\pi}{2}\right) + \frac{\cos(-\pi)}{2}\right)\right]$

$= \frac{1}{2} + \frac{1}{4} - \left(-1 - \frac{1}{2}\right)$

$= 2\frac{1}{4}$ units^2

e) i)



$\sin\theta = \frac{9}{L_1}$

$L_1 = \frac{9}{\sin\theta}$

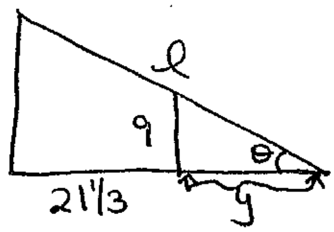
$\cos\theta = \frac{21\frac{1}{3}}{L_2}$

$L_2 = \frac{21\frac{1}{3}}{\cos\theta}$

$L_2 = \frac{64}{3\cos\theta}$

$L = L_1 + L_2$

$= \frac{9}{\sin\theta} + \frac{64}{3\cos\theta}$



$\tan\theta = \frac{9}{y}$

$y = \frac{9}{\tan\theta}$

$\cos\theta = \frac{21\frac{1}{3} + y}{l}$

$l = \frac{21\frac{1}{3} + y}{\cos\theta}$

$l = \frac{1}{\cos\theta}\left[21\frac{1}{3} + y\right]$

$= \frac{1}{\cos\theta}\left[\frac{64}{3} + \frac{9}{\tan\theta}\right]$

$= \frac{64}{3\cos\theta} + \frac{9}{\frac{\sin\theta \times \cos\theta}{\cos\theta}}$

$= \frac{64}{3\cos\theta} + \frac{9}{\sin\theta}$.

ii)

$L = 9(\sin\theta)^{-1} + \frac{64}{3}(\cos\theta)^{-1}$

$\frac{dL}{d\theta} = -9(\sin\theta)^{-2}\cos\theta - \frac{64}{3}(\cos\theta)^{-2}(-\sin\theta)$

$= -\frac{9\cos\theta}{\sin^2\theta} + \frac{64\sin\theta}{3\cos^2\theta}$

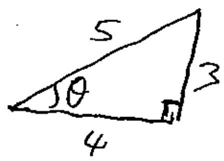
$\frac{dL}{d\theta} = 0, \quad 0 = -\frac{9\cos\theta}{\sin^2\theta} + \frac{64\sin\theta}{3\cos^2\theta}$

$27\cos^3\theta = 64\sin^3\theta$

$\tan^3\theta = \frac{27}{64}$

$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$



θ	$35°$	$\tan^{-1}(\frac{3}{4})$	$40°$
$\frac{dL}{d\theta}$	-0.921	0	1.620

Minimum $L = \frac{9}{\sin\theta} + \frac{64}{3\cos\theta}$

$= 9 \times \frac{5}{3} + \frac{64}{3} \times \frac{5}{4}$

$= 41\frac{2}{3}$ m